



An energy model for projectile sound

K.-W. Hirsch, Institut für Lärmschutz, Arnheimer Straße 107, D-40489 Düsseldorf

Abstract

Supersonic projectiles generate an acoustical shock wave along their trajectory. This projectile sound is only audible in the Mach area. The geometry of this area depends on the projectile speed relative to the speed of sound and on the decrease of the projectile speed along the trajectory. At some distance from the projectile, the shape of the waveform is the typical N-wave.

In 1950 and 1952, Witham published two papers on the prediction of the sound pressure of projectile sound including the non-linear effects. The pressure prediction depends on the diameter, length and shape of the projectile and on the local Mach number. As a consequence of non-linearity, the spectral energy content is not constant but depends on distance.

For large area, multiple source noise contour maps, this model leads to long calculation times and - due to some limitations in the model - generates prediction errors in those cases where the projectile speed becomes subsonic along its trajectory. Therefore, an energy model for projectile sound was developed to overcome these problems. This energy model assumes that the source of the projectile sound is the local loss of kinetic energy. A fraction of that energy loss is radiated as sound energy into the direction determined by the local Mach number. For distances far enough to apply linear acoustics, this model predicts the free field sound exposure level and a constant time duration of the N-wave. The paper introduces this model and compares the result to the non-linear pressure model.

Introduction

Shooting noise can be the result of three independent sound sources: the muzzle blast, the projectile sound and - with mostly military applications - the demolition blast at the target. As long as shooting noise is measured for noise assessment purposes, the contributions of these three sources make-up the receiver level for a single shot event. In Germany for example, the regulation of the VDI 3745, Part I, prescribes such a measurement for small arms. For prediction purposes of shooting noise however, the different sound sources must be treated separately because the description of the source and the propagation is different for each source. This paper deals with the description of projectile sound.

In 1950 and 1952, Witham published two papers on the prediction of the pressure of the acoustical shock wave generated by supersonic projectiles /1/, /2/. The pressure prediction depends on the local Mach number, on the diameter, on the length and - in a rather sophisticated way - on the shape of the projectile. This model includes the non-linear effects close to the source and is widely used to predict peak pressures for example for hearing

protection purposes, to assess the effect of supersonic flying air crafts on buildings, e.g. /3/, and in the context of military reconnaissance, e.g. /4/ or /5/. It was also already used for the prediction of shooting noise from large military guns, e.g. /6/.

Van den Berg et al., /7/, applied Witham's results to develop a sophisticated model to predict the projectile sound from small arms in terms of sound exposure level. They define a source level, a non-linear term, a correction for geometric spreading in the way that is typically used in the context of noise prediction models. The model also includes a correction for the loss of coherence during propagation which is special to projectile sound, /8/.

Though the generation and propagation of projectile sound seems to be well understood, there is a need for a more simple 'estimation' model for the following reasons:

- All models based on Witham's results are non-linear with respect to acoustical pressure. As a consequence, the spectral energy content is not constant but depends on distance. Therefore, guidelines for the calculation of sound propagation, e.g. ISO 9613-2 - in particular with respect to ground correction and shielding –, are not directly applicable.
- Due to some limitations in Witham's equations, there are prediction errors in those cases where the projectile speed becomes subsonic along the trajectory. This normally occurs for instance with pistols or shot guns.
- At present, the ISO/TC 43/SC 1/JWG51 is compiling an new international standard (ISO 17201) to establish rules for the prediction of noise from civil shooting. The ISO 17201 will have 5 parts; part 1 proposes a method to describe the source strength of the muzzle blast, part 2 will give guidance to estimate source parameters from poor input data, part 3 deals with propagation of the coherent blasts, part 4 will describe a source model for projectile sound and, finally, part 5 will collect assessment procedures. The JWG51 will follow the ideas of /7/ for part 4. However, in the context of this standard also a simple model is needed to estimate projectile sound from poor input data.
- For large area, multiple source noise contour maps, the models based on Witham's results lead to long calculation times. For noise management purposes at military training areas for instance faster procedures are necessary.

The energy model

Basic idea

Textbooks normally introduce 'sonic boom' as a line source having cylindrical spreading of energy. As a consequence, a constant Mach angle defines a Mach area around the trajectory from source to target where sound is propagating into a constant direction. For 'sonic boom' from ballistic projectiles not propelled along their trajectory, this is basically not true. The kinetic energy of the projectile translational and rotational is the only source where the acoustical energy of projectile sound can come from. Hence, deceleration is imperative, see fig. 1. As a consequence, there is not a constant Mach angle and not a simple geometric spreading. That means, even a simple model needs to follow the complicated rules of geometry discussed later. However, introducing the loss of energy per unit length on the trajectory will lead to a more simple view on the source strength of projectile sound.

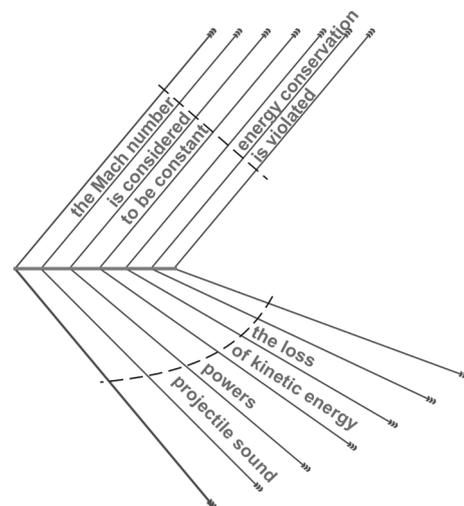


Fig. 1 Projectile sound field of a ballistic projectile
upper half: constant speed
lower half: decreasing speed

Source energy density of projectile sound

The sound is not the only reason for deceleration. Air friction and displacement will also consume energy, for example. These phenomena normally depend for instance on the instantaneous projectile speed and gyration and on the shape of the projectile. Assuming that the relation of the fractions of all losses is constant yields that the acoustical energy radiated from a unit length of the trajectory, the sound source energy density, is proportional to the energy loss per unit length of the translational kinetic energy.

$$e_{ac} = f_{ac} \hat{e} \quad (1)$$

In eq. 1, let denote \hat{e} the specific energy loss per unit length, let denote f_{ac} the acoustical efficiency and let denote e_{ac} the acoustical source energy density.

There are several ways to calculate or to estimate \hat{e} . Some ammunition catalogues directly provide this information or they include tables for the decay of projectile speed versus shooting distance; normally the mass is also mentioned so that both kinetic energy and speed is known as input data. For all military rifles and guns this information is well-known.

The acoustical efficiency f_{ac} is the first and only free parameter in the model; f_{ac} depends on the shape of the projectile on the instantaneous projectile speed and so on. As a first approach, this paper assumes $f_{ac} = 0.25$, constant along the trajectory, and for all projectiles. This setting is supported by comparisons with the more sophisticated pressure model for three cases: a howitzer 155 mm projectile, a G3 military rifle 7.62 projectile and for a shot gun. This comparison is discussed later and in more detail in /9/. Nevertheless, this parameter can be adopted to measurements within this model.

Geometrical spreading

The ‘geometrical spreading’ is a pure geometrical function to describe the increasing area, the same portion of energy is passing through during propagation. Considering a straight trajectory segment of length l , this area is a cylindrical skin around the section under consideration for very small distances. For greater distances this area S will grow and becomes a certain wedge, that is rotational symmetric around the line of fire.

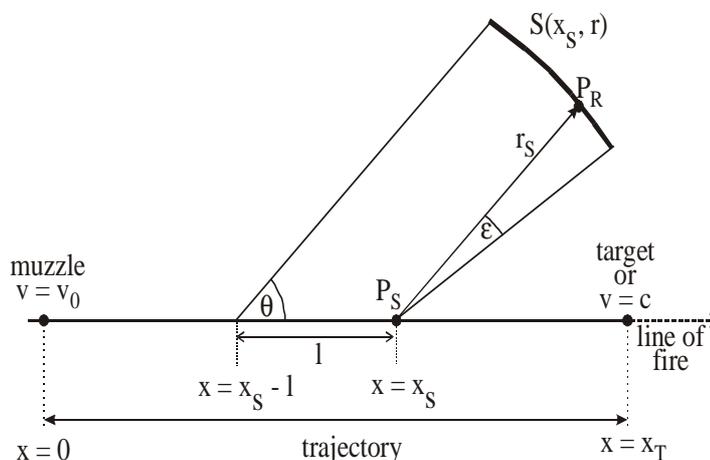


Fig. 2 Calculation of geometric spreading

Fig. 2 shows one way to approximately calculate S for an arbitrary receiver point P_R . This way aims on numerical calculations, in particular. The projection of S is a curved line split into two sections, each assumed to be straight. The section left to P_R of S in fig. 2 represents the cylindrical spreading. This contribution to S will increase linearly with the distance r_s . The section right to P_R represents a kind of spherical spreading increasing with the distance square due to change of the projectile speed and the change of the Mach-angle, respectively.

This visual way of finding a formula for the geometric spreading makes clear that the sound at P_R is generated along l . It introduces geometrical parameters that are easily linked to the kinetic parameters of the projectile along its trajectory. This approach also holds for the

last segment of the projectile's trajectory before it becomes subsonic. However, it only holds if the l is sufficient small. Numerical calculations show that $l = 1$ m is a reliable setting for most cases.

All necessary geometric parameters are defined in fig. 2. The x -axis is the line of fire. Fig. 2 assumes that the small trajectory section l generates the projectile sound through $S(x_S, r)$. Let denote θ the angle of sound radiation at $x = x_S - l$ (which is the 90° -complement of the Mach-angle and depends on the local speed v). Let denote ε the decrease of that angle up to the end of l . The so-called geometrical source point P_S for the receiver point P_R is determined by the condition that P_R lies on the line that intersects the trajectory with θ ; r_S is the so-called propagation distance.

Then $S(x_S, r_S)$ is approximately

$$S_S(r_S) = 2\pi l^2 \left[\sin^2 \theta_S \left(\frac{\cos \theta_S}{2} + \frac{r_S}{l} \right) + \frac{r_S^2}{l^2} \sin \left(\theta_S - \frac{\varepsilon_S}{2} \right) \sin \varepsilon_S \right] \quad (2)$$

$$\theta_S = \arccos \left(\frac{c}{v(x_S - l)} \right) \quad (3)$$

$$\varepsilon_S = \arccos \left(\frac{c}{v(x_S - l)} \right) - \arccos \left(\frac{c}{v(x_S)} \right) \quad (4)$$

The acoustical energy density e at P_R is the radiated energy from l divided by the appropriate area S_S , (neglecting any additional influences on the propagation of sound like air absorption).

$$e(P_R) = \frac{e_{ac}(l) l}{S_S(r_S)} \quad (5)$$

Sound exposure

The goal of this simple model is to predict the sound exposure SE at an arbitrary receiver point P_R . Eq. 6 defines the sound exposure SE at P_R and at distance r_S , respectively

$$SE_S(r_S) = \int_{\text{projectile sound}} p^2(r_S, t) dt \quad (6)$$

Let denote p the sound pressure and t the time. If the rules of linear acoustics apply, the sound pressure at P_R for free field propagation SE at r_S yields to be

$$SE_S(r_S) = \rho c \frac{e_{ac}(l) l}{S_S(r_S)} \quad (7)$$

because the energy density is the time integral of the intensity over the whole projectile sound event. The intensity - pressure times particle velocity - can be replaced by pressure square times the impedance ρc . Let denote ρ then density of air and c the speed of sound.

Limitation due to non-linearity, N-Wave duration

The prediction of sound exposure only holds for such distances where the sound propagation follows the rules of linear acoustics. The non-linear models, see e.g. /7/, predict the peak pressure of a projectile in dependence on the propagation distance. Assuming that for peak pressures lower than $P_{lin} = 100$ Pa linear acoustics apply, these models yield a critical distance r_{lin} . If r_S is smaller than r_{lin} the peak pressure exceeds P_{lin} . This procedure yields

$$r_{lin} = 4480 \frac{M}{(M^2 - 1)^{\frac{5}{12}}} \sqrt[3]{\frac{d^4}{l}} \quad (8)$$

In eq. 8, let denote d the diameter and l the (effective /7/) length of the projectile and let denote M the Mach number. So far shooting noise prediction is concerned the distance between P_S and P_R normally exceeds r_{lin} .

In order to complete the energy model, eq. 9 estimates the 'linear' time duration of the projectile sound (N-wave) following the same procedure and assumptions as for r_{lin} .

$$t_{clin} \approx 15 \frac{M}{c} \sqrt[3]{\frac{d^4}{l(M^2 - 1)}} \quad (9)$$

The 'linear' time duration t_{clin} also clearly defines the spectrum of the sound because the shape of the pressure time history of the projectile sound is a clear determined N-wave if that duration is known.

Comparison

Tab. 1 shows the parameters for three shots used here to compare the results of the 'energy model' to the 'pressure model', /7/. The following calculations do not consider air absorption. The segment l to evaluate the geometric spreading for the energy model was set to be 0,01 m for all samples. The line of fire is always straight up to a (theoretical) target at 5000 m. For the shot gun, the level was increased by 20 dB to represent energy addition for 100 pellets. /9/ presents a more detailed comparison of both models.

weapon	muzzle speed	decay of speed	mass	diameter	effective length
	m/s	(m/s)/m	g	mm	mm
howitzer 155	560	0,1	40k	155	150
rifle 7.62	780	0,8	8	7,62	7
shot gun	420	10	0,12	4,0	2

Tab 1 Input parameters for the shots

Fig. 3 compares the results of the projectile sound on a straight line beginning 1 m in front of the muzzle at 30°. For all samples, the non-linear increase of the levels close to the trajectory is obvious. At larger distances the prognosis of the pressure are lower than from the energy model due to the still effective non-linear term in the pressure model. For the howitzer shot at 8000 m and the shot gun at 80 m the coherence correction comes into play. For the range of interest with respect to noise prediction the energy model comes close to the pressure model.

Fig. 4 compares the results on a half circle with a radius of 250 m around the point 1 m in front of the muzzle. For the howitzer and the rifle shot both models agree sufficiently for noise prediction purposes. The pellets of the shot gun become subsonic. Therefore the results are different. For a 20° segment in front of the shot gun the prediction of the pressure model is missing due to the restriction in that model ($M > 1.01$).

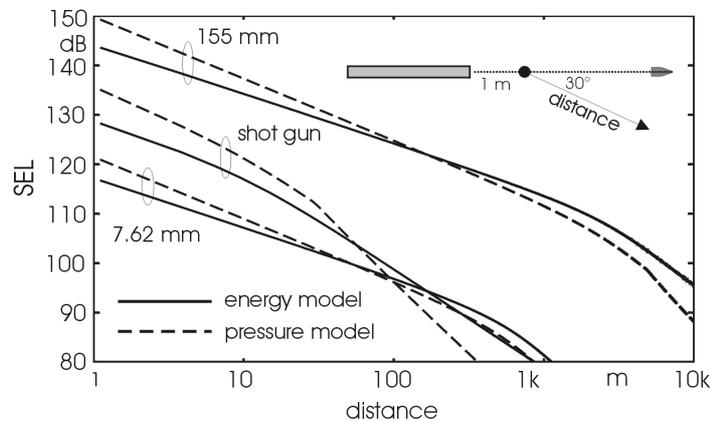


Fig. 3 Comparison on a line

Conclusions

This paper proposes an estimation model for projectile sound that

- can serve as an estimation model in ISO 17201,
- can make predictions for projectiles becoming subsonic,
- is a linear approach and therefore compatible to ISO 9613-2,
- is applicable for large noise contour maps.

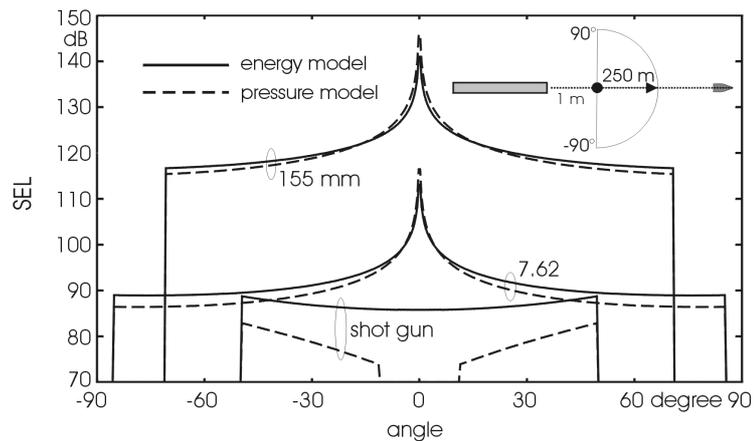


Fig. 4 Comparison on a half circle

The comparison of the proposed energy model to the pressure model yields that both models are in good agreement at those distances that are important for the prediction of shooting noise.

Acknowledgements

The German Ministry of Defense supports these investigations.

References

1. Witham, G.B.: "The behaviour of a supersonic flow past a body of revolution far from the axis", Proc. R. Soc. London, Ser. A261, (1950)pp.89-109
2. Witham, G.B.: "The flow patterns of a supersonic projectile", Commun. Pure Appl. Math. V(1952)pp.301-348
3. Koch, H.W.; Weber, G.: "Flugzeugknalle und ihre Wirkung auf Gebäude", Z. Die Bautechnik, 7(1970)238ff
4. Naz, P.; Parmentier, G.: "Akustische Erkennung von Heckenschützen" in Anwendungen der Akustik in der Wehrtechnik, Kurt Nixdorff (Editor), September 1998
5. Naz, P.; Kronenberger, G.; Burde, E.; Parmentier, G.: "Messungen von Waffenknall, angewendet auf die Detektion von Heckenschützen, Teil 1: Physikalische Grundlagen", Technischer Bericht S-RT 902/98 des Deutsch-Französischen Forschungsinstitutes, ISL
6. Brinkmann, H.: "Messungen für Lärmkataster WBV", Frequenzanalysen von Knallen des KPz Leopard 2", Bericht der Erprobungsstelle 91 der Bundeswehr, Meppen, August 1968
7. van den Berg, F.H.A; Noordhoek, I.M., Otter, G.C.J.: "A practical method for calculating the long-range sound propagation of projectile noise", Proceedings Internoise 2000, Nice, S.791-794
8. Salomons, E.M.: "A coherent line source in a turbulent atmosphere", J. Acoust. Soc. Am. 105 (2), pp 652-657
9. Hirsch, K.-W., Trimpop, M.: "Vergleich verschiedener Ansätze zur Prognose der Schallimmission von Geschosknallen", DAGA 2001, Hamburg, to be printed